Response of massive bodies to gravitational waves

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Abstract

The response of a massive body to gravitational waves is described on the microscopic level. The results shed a new light on the commonly used oscillator model. It is shown that apart from the non-resonant tidal motion the energy transfer from a gravitational wave to an electromagnetically coupled body is in general restricted to the surface, whereas gravitational coupling gives rise to bulk excitation of quadrupole modes, but several orders of magnitude smaller. A microscopic detector making use of the effect is suggested.

1 Introduction

Gravitational waves were already considered by Einstein as the wave solutions of the linearized field equations of gravity. There is indirect evidence of their existence through systems of binary pulsars that loose energy in form of gravitational radiation [1], their direct experimental measurement presently is one of the most challenging tasks in gravitational physics. Very sensitive detectors operating at the quantum limit are needed to detect directly gravitational waves from cosmic events such as collapsing or colliding star systems. There are basically two different types of detectors: resonant mass antennas based on the resonant excitation of quadrupole-type modes of a appropriately chosen massive body, like the bar detectors conceived by Weber [2], and laser interferometric devices that detect the direction-dependent variation of the proper distance between the mirrors of a Michelson interferometer [3]. Detectors of both types are presently under construction [4].

Commonly a resonant mass antenna is described in Riemannian normal coordinates with respect to its center of mass, the proper frame of reference (PFR). The detector is analyzed in term of normal modes, idealized by a spring that couples two masses [2], the resonant energy input is calculated. The intention of the work presented in this article was twofold: First, to validate the results obtained from the normal mode model by microscopic considerations, second to give a complementary description of the detector in the reference system of the wave, in which the linearized solutions of the Einstein field equations are computed. Our model is based on the local properties of a detector in terms of the fundamental binding forces, electromagnetic and other, which we consider in both the PFR and the wave system. Analyzing the deviations from the tidal motion, we find that the energy input from gravitational waves on an electromagnetically coupled massive body is restricted to the surface of the body, whereas gravitational coupling leads to true bulk excitation of quadrupole modes. This result does not contradict the normal mode picture at all, rather it presents a complementary viewpoint that has eluded the normal mode analysis. The reason is that though the energy input into any single mode is nonlocal, in the special case of gravitational waves the superposition of all excited modes describes a localized excitation. Based on our observations we propose a new, microscopic type of detector.

2 Frames of reference

There exist two frames of reference that can be used for the analysis of gravitational waves and gravitational wave detectors. The natural system to study the waves is a perturbed Minkowski system for which the linearized Einstein field equations are solved. In this reference system the plane wave is gauged, conventionally a transverse-traceless (TT) gauge is chosen [2]. On the other hand, the use of the PFR system with Riemannian normal coordinates with respect to the center of mass is the natural system for the study of a detector. For our purpose to study of the detector on the microscopic level, both system have advantages and disadvantages. In the PFR system we have common argument [5] that in the metric of the gravitational wave field leads to variations of the electromagnetic field of order

$$\delta A/A \sim \left(L^2/\lambda^2\right) h^{TT} \tag{1}$$

where L is the distance from the origin, λ and h^{TT} the wave length and amplitude of the gravitational wave, respectively, whereas the tidal gravitational forces lead to displacements of the constituent particles of the body that in turn lead to variations of the electromagnetic field of order

$$\delta A/A \sim h^{TT} \tag{2}$$

so that the metric effect can be neglected. This means that in the PFR system the unperturbed solutions of the Maxwell equations can be used. On the other hand, in the PFR system we have to deal with the general problem of general relativity that local energy densities have no invariant meaning, so that we cannot easily control the exchange of energy between the wave and the detector. Here the TT system has the advantage that we can employ a Hamiltonian description of the constituent particles, and because the system is in principle a special relativistic one, we have the standard laws of energy-momentum conservation. Therefore the TT system proves useful for our considerations. Because considerations in both coordinate system contribute to the understanding of the detector response, we formulate our results in both systems.

We first show that in the TT system no energy is transferred to a system of non-interacting particles; in the PFR system this corresponds to the well-known quadrupole type oscillations. Next we extend the discussion to electromagnetically coupled systems, where we can use the standard Coulomb force in the PFR system, whereas we have to solve the metric Maxwell equations in the TT system. Finally we discuss a gravitationally coupled system and show that there exists a decisive difference in the response of the detector which is due to the different nature of the fundamental forces.

3 Particle Motion

A point-like test mass with electric charge e can be described in the presence of electromagnetic fields and gravitation by the Hamiltonian [7]

$$H = c\sqrt{\left(p_\mu - eA_\mu\right) g^{\mu\nu} \left(p_\nu - eA_\nu\right)} \tag{3}$$

where we use coordinates $x^\mu, \mu = 0, 1, 2, 3, 4$ for space-time with metric $g_{\mu\nu}(x^\kappa)$ and signature $+ - - -$, p_μ are the momentum coordinates in the cotangent space, and $A_\mu(x^\kappa)$ is the electromagnetic four-potential. We also use the notation (ct, x, y, z) in an obvious manner. The evolution parameter will be denoted by τ. The Hamiltonian is conserved, $\partial H/\partial\tau = 0$, it represents the rest mass $m = H/c^2$of the particle.

The canonical equations of motion are given by

$$\dot{x}^\mu = \frac{\partial H}{\partial p_\mu}, \qquad \dot{p}_\mu = -\frac{\partial H}{\partial x^\mu}. \tag{4}$$

The constancy of H is equivalent to $\dot{x}^\mu g_{\mu\nu}\dot{x}^\nu = c^2$ for any trajectory, thus the evolution parameter is the proper time.

A gravitational wave propagating in z-direction with $+$ and $\times$ polarization modes is described in TT gauge by the metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1+f_+(ct-z) & f_\times(ct-z) & 0 \\ 0 & f_\times(ct-z) & -1-f_+(ct-z) & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \eta_{\mu\nu} + h_{\mu\nu} \tag{5}$$

where $h_{\mu\nu}$ is a small perturbation of the Minkowski metric $\eta_{\mu\nu}$ [2]. This perturbation acts as classical, special relativistic field, so that the system can be treated within special relativity, except that the interpretation of the energy must be treated with care [6]. In the absence of an electromagnetic field we obtain for this metric a Hamiltonian that leads to four conserved quantities: H, p_x, p_y, and $p_0 + p_3 = E/c + p_z$ where E is the energy of the particle in the sense of special relativity. Note that $p_3 = -m\dot{z}$, so that the difference between the energy and the conventional z-momentum is conserved. This is natural, since the gravitational wave not only carries energy E_w, but also momentum P_w with the relation $E_w = c\,|P_w|$ that holds for all massless objects in special relativity. The exchange ΔE of energy between the wave and a test mass thus is always accompanied with an exchange ΔP of momentum:

$$\Delta E = c\Delta P. \tag{6}$$

The existence of four conserved quantities now allows us to integrate the equations of motion completely:

$$\begin{array}{ll} \dot{x}^0 = \frac{1}{m}p_0 & \dot{x}^2 = \frac{1}{m}\left(g^{21}p_1 + g^{22}p_2\right) \\ \dot{x}^1 = \frac{1}{m}\left(g^{11}p_1 + g^{12}p_2\right) & \dot{x}^3 = -\frac{1}{m}p_3 \\ p_1 = const & p_2 = const \end{array} \tag{7}$$

$$\begin{aligned} p_0 &= \frac{1}{2}(p_0+p_3) + \frac{1}{2}\frac{m^2}{(p_0+p_3)}\left(c^2 - \frac{1}{m^2}p_a g^{ab} p_b\right) \\ p_3 &= \frac{1}{2}(p_0+p_3) - \frac{1}{2}\frac{m^2}{(p_0+p_3)}\left(c^2 - \frac{1}{m^2}p_a g^{ab} p_b\right) \end{aligned} \tag{8}$$

where we use the indices a, b for a summation over $1, 2$ only. Let us consider a wave pulse. We denote the initial conditions before the arrival of the pulse by $\bar{p}_\mu$, and have

$$\begin{aligned} p_0(\tau) &= \bar{p}_0 - \frac{1}{2}\frac{p_a h^{ab} p_b}{\bar{p}_0 + \bar{p}_3} \\ p_3(\tau) &= \bar{p}_3 + \frac{1}{2}\frac{p_a h^{ab} p_b}{\bar{p}_0 + \bar{p}_3} \end{aligned} \tag{9}$$

where $h^{ab} = g^{ab} - \eta^{ab}$. Thus after the pulse, where the perturbation h is zero again, the particle has the same four-momentum as before, and the only possible effect is a displacement of the straight trajectory after the pulse from the one before the pulse. If a particle is initially at rest, it stays at rest in this reference frame. Two particles that are at rest relative to each other, remain at rest relative to each

other, though the proper distance between them changes with the wave amplitude. Thus free test particles do not take up energy from a gravitational wave. When we restrict this statement to a comparison of the energy before and after a wave pulse or wave train, we are moreover free from the ambiguity of the energy definition in general relativity.

We now look at this result in the PFR system. In the PFR system particles do not stay at rest in the wave field, but move under tidal accelerations. In PFR coordinates $\left(\hat{t}, \hat{x}, \hat{y}, \hat{z}\right)$ the tidal accelerations produced by the wave are [2]:

$$\frac{d^2\hat{x}^a}{d\hat{t}^2} = -\hat{R}_{a0b0}\hat{x}^b = \frac{1}{2}\frac{d^2 h_{ab}}{d\hat{t}^2}\hat{x}^b \tag{10}$$

For a particle that is initially at rest at $\hat{x}^{a(0)}$ before a wave pulse arrives, the solution is simply given by

$$\hat{x}^a\left(\hat{t}\right) = \hat{x}^{a(0)} + \frac{1}{2}h_{ab}\left(\hat{t}\right)\hat{x}^{b(0)}. \tag{11}$$

(Note that $\hat{R}_{a0b0}$ and $\hat{R}^a{}_{0b0}$ differ only by terms of order $O\left(h^2\right)$). This is the well-known quadrupole-like tidal motion of a system of particles around the origin of the frame of reference. The coordinate distance of the particles varies according to the changing metric distance between them. We can ascribe a standard kinetic energy to this motion, but there will be additional contributions to the conserved energy from the metric. Our analysis in the TT system has shown that the energy of the wave does not vary in this case. The coordinate transformation from the TT to the PFR system is time-dependent, so that the energy conservation in the TT system translates into a more complicated conservation law in the PFR system.

Because free particles do not effectively take up energy from a gravitational wave, we have to take the coupling between particles into account in order to describe the response of a detector. The coupling is basically of electromagnetic nature in small massive bodies, but may also be of gravitational nature in large bodies. Because the effects of the gravitational wave are so small, massive detectors must be cooled to zero temperature as near as possible. Thus the ground state of a body where the constituent atoms or ions are at rest relative to each other except for quantum effects will serve as an appropriate model. Rotational motion with respect to the TT frame of reference must be taken into account. Internal motion, thermal or otherwise, of the atoms leads to forces of order $m\Delta v\partial h/\partial t$; we do not consider this kind of phonon-graviton interaction in this article, because it is the idea of the Weber detector to excite the quadrupole modes, not to enhance already excited modes.

4 Electromagnetic Field

We first look at the Coulomb potential generated by a charge that resides in the field of the wave. In the PFR system we have the above cite argument that the metric perturbations of the electromagnetic potentials can be ignored. Thus, for the description of an electromagnetically bound solid body, we have to take only the Coulomb forces into account, and can ignore contributions from magnetic fields and electromagnetic radiation. In the PFR system the coordinate distance agrees with the metric distance up to order $O(h)$. Therefore we can state in an invariant manner, that the Coulomb potential depends only on the metric distance of the particles. We now show how this translates into the TT system.

We assume that the wave field is slowly varying and the velocity of the charge relative to the source of the fields is so small that magnetic fields can be ignored.

The corresponding equation to solve for the electromagnetic potential generated by a charge q_s at rest at $x = y = z = 0$ is

$$\partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu A^0 = \frac{q_s}{\varepsilon_0} \delta^3(x, y, z). \tag{12}$$

Our considerations in the PFR system give rise to the following ansatz:

$$A_s^0(\vec{r}, ct - z) = \frac{q_s}{4\pi\varepsilon_0 r}, \quad \vec{r} = \left(x^i\right)_{i=1,2,3}, \quad r^2 = -x^i g_{ij} \left(ct - z\right) x^j \quad . \tag{13}$$

where the Euclidean coordinate distance from the source is replaced by the metric distance. We verify that equation (12) is satisfied to order $O(h)$. For simplicity only we consider a wave with the $+$-mode only where

$$r^2 = x^2(1 - f_+ \left(ct - z\right)) + y^2 \left(1 + f_+ \left(ct - z\right)\right) + z^2. \tag{14}$$

Using $\sqrt{-g} = 1 + O\left(h^2\right)$, leaving out terms of order h^2 and higher, we obtain :

$$\begin{aligned}
\partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu \frac{1}{r} &\cong \left(c^{-2}\partial_t^2 - \frac{1}{1 - f_+}\partial_x^2 - \frac{1}{1 + f_+}\partial_y^2 - \partial_z^2\right)\frac{1}{r} \\
&= c^{-1}\partial_t \left(\frac{x^2 - y^2}{2r^3} f_+'\right) + \partial_x \frac{x}{r^3} + \partial_y \frac{y}{r^3} - \partial_z \left(-\frac{z}{r^3} - \frac{x^2 - y^2}{2r^3} f_+'\right) \\
&= \nabla \frac{\vec{r}}{r^3} + \frac{3\left(x^2 - y^2\right)^2}{4r^5}\left(f_+'\right)^2 + \frac{x^2 - y^2}{2r^3} f_+'' - \frac{x^2 - y^2}{2r^3} f_+'' + f_+' \partial_z \frac{y^2 - x^2}{2r^3} \\
&= 4\pi\delta\left(r\right) + \frac{3\left(y^2 - x^2\right)^2}{4r^5}\left(f_+'\right)^2 + f_+' \partial_z \frac{y^2 - x^2}{2r^3}
\end{aligned} \tag{15}$$

Now the first of these two remaining terms is of order $O\left(h^2\right)$ and can thus be ignored, the second is proportional to the spatial derivative of the wave field, which can be ignored for detectors that are small against the wave length [2]. Thus the ansatz (13) solves equation (12) except for terms of order $O\left(h^2\right)$ or proportional to $\partial_z h$. In the same sense the Lorentz gauge holds. The result thus agrees with that in the PFR system: The Coulomb potential depends only on the metric distance.

In the following we assume that this principle may also by extended to other, phenomenological potentials, because the time scale set by the gravitational waves is by far larger than that of the induced changes of all other fundamental interactions. So in the TT system the Coulomb force between two charged particles varies in phase with the gravitational wave. It is not hard to see that the local energy density of the electromagnetic field, though time-dependent, is only relocated, so that the integrated energy density does not change to first order in h, implying that radiation effects are at most of second order in h, in agreement with the situation in the PFR system. On the other hand accelerated, moving charges produce magnetic fields and electromagnetic radiation, but both effects can be ignored for the analysis of weak mode excitation in a solid body.

5 Many particles

The main point of this work is to show that a detailed microscopic, many-particle model of the detector exhibits two aspects that elude the spring model and the normal mode analysis. The first aspect is that the input of energy from the wave to the detector occurs only at the surface of the detector. This effect can already be modelled with a linear chain , as done in the next subsection. The second aspect is the dependence on the nature of the fundamental forces that stabilize a body: electromagnetic and gravitational binding forces lead to different mode structure of the responde, as shown subsequently.

5.1 Linear chain model

In the PFR system N classical particles with coordinates $\vec{r}^{(s)} = (\hat{x}, \hat{y}, \hat{z}), \alpha = 1, ..., N$ obey the equations of motion

$$m_i \frac{d^2}{d\hat{t}^2} \vec{r}^{(s)} = \sum_{s' \neq s} \vec{F}_{s's} \left(\vec{r}^{(s')}, \vec{r}^{(s)} \right) + \frac{m_i}{2} \frac{d^2 h}{d\hat{t}^2} \cdot \vec{r}^{(s)} \tag{16}$$

where for simplicity the dot denotes the multiplication of the 3×3 matrix h_{ij} with the vector $\vec{r}^{(s)}$, and $F_{s's}$ represents the fundamental forces between two particles. Let us consider the special model of a linear chain in $\hat{x}$-direction with $N = 2n$ equal particles numbered by $s', s = -n...n$, coupled by nearest-neighbor forces

$$F_{s's} \left(\hat{x}^{(s')}, \hat{x}^{(s)} \right) = -\omega_0^2 \left(\hat{x}^{(s')} - \hat{x}^{(s)} - l_0 \right) \text{ for } |s' - s| = 1 \tag{17}$$

and a wave with $+$-mode only, so that the equations of motion are

$$\frac{d^2}{d\hat{t}^2} \hat{x}^{(s)} = -k \sum_{\beta = \alpha \pm 1} \left(\hat{x}^{(s)} - \hat{x}^{(s')} \pm l_0 \right) + \frac{1}{2} \frac{d^2 f_+}{d\hat{t}^2} \hat{x}^{(s)} \tag{18}$$

where $k = \omega_0^2 / m$. Without the tidal acceleration, the equilibrium positions are $\hat{X}^{(s)} = s l_0$, relative to the center of mass that is identical with the origin of the coordinate system. We are now interested in the deviation from the tidal motion (**??**) because the tidal motion itself will be in general too small to be observed itself. We set

$$\xi^{(s)} = \hat{x}^{(s)} - \hat{X}^{(s)} \left(1 + \frac{f_+}{2} \right) \tag{19}$$

leading to

$$\begin{aligned}
\ddot{\xi}^{(s)} &= -k \sum_{s' = s \pm 1} \left(\xi^{(s)} - \xi^{(s')} - \left(\hat{X}^{(s)} - \hat{X}^{(s')} \right) \left(1 + \frac{f_+}{2} \right) \pm l_0 \right) + \frac{1}{2} \frac{d^2 f_+}{d\hat{t}^2} \left(\hat{x}^{(s)} - \hat{X}^{(s)} \right) \\
&= -k \sum_{s' = s \pm 1} \left(\xi^{(s)} - \xi^{(s')} \pm l_0 \frac{f_+}{2} \right)
\end{aligned} \tag{20}$$

where we have omitted the term $\frac{1}{2} \frac{d^2 f_+}{d\hat{t}^2} \left(\hat{x}^{(s)} - \hat{X}^{(s)} \right)$ because we consider deviations from the tidal motion that are proportional to f_+ itself, as induced by the wave, and thus this term is of order $O\left(h^2\right)$. Now the following happens: the terms $l_0 \frac{f_+}{2}$ in (20) cancel for all particles that have two neighbors, only for the particles at the ends of the chain they do not. The reason is that under the tidal motion the distances between pair of neighboring particles remains constant, thus the induced additional forces from the left and right cancel; the pattern of the tidal acceleration comes very close to a null mode. So we can rewrite

$$\ddot{\xi}^{(s)} = -k \sum_{s' = s \pm 1} \left(\xi^{(s)} - \xi^{(s)} \right) - k l_0 \frac{f_+}{2} \left(\delta_{s,n} - \delta_{s,-n} \right). \tag{21}$$

These equations have a clear interpretation: The deviation from the tidal motion is driven by an effective force that applies to the ends of the chain only. Or, we can state that it suffices to apply additional forces $\mp k l_0 f_+ / 2$ to the ends of the chain in order to suppress any deviations from the tidal motion. In this case the tension of the chain varies uniformly along with the wave strength, but no work is done at all against the internal coupling forces.

Regarding the energy we have to be careful to use standard expressions, because there are metric contributions, as we have seen above, but we can certainly state that apart from the tidal motion energy is transferred to the chain only locally at the ends. For resonance cross terms in the kinetic energy between the tidal motion and the deviations from it play no role, so that $\sum m\dot{\xi}^2/2$ describes the kinetic energy of interest. Nevertheless this local picture does not contradict the normal-mode analysis in any way. We still have the possibility to decompose the perturbing force into normal modes and derive equations for the driving of these modes in the standard way. The fundamental mode of the chain is driven strongest, but all other symmetric modes are also excited. It is the superposition of all these modes that gives rise to the local excitation of the chain. Only refining the spring model to a linear chain model could exhibit this property.

Because we see from (21) that the effective force depends on the microscopic property of the chain in form of the equilibrium distance l_0, it is necessary to analyze a detector on the basis of a more realistic microscopic model that yield information on the strength of the driving forces. It turns out that this is also necessary because the difference between gravitational and electromagnetic coupling can only then be shown.

5.2 Microscopic forces in the PFR system

For a more realistic model we can use eq. (16) with the fundamental Coulomb, gravitational, and repulsive short-range forces inserted. We assume that all these forces depend only on the difference vectors $\vec{r}^{(\beta)} - \vec{r}^{(\alpha)}$ and stable equilibrium positions $\vec{R}^{(\alpha)}$ exists. Again we look at the deviation from the tidal motion induced by these forces. The transformation

$$\vec{r}^{(\alpha)} = \left(1 + \frac{1}{2}h\right) \cdot \vec{\rho}^{(\alpha)} \tag{22}$$

now leads, assuming small deviations from equilibrium,

$$\vec{\rho}^{(\alpha)} = \vec{R}^{(\alpha)} + O\left(h\right), \tag{23}$$

to

$$\begin{aligned} \frac{d^2}{d\hat{t}^2}\vec{\rho}^{(\alpha)} &= \sum_{\beta\neq\alpha} \vec{F}_{\beta\alpha}\left(\left(1 + \frac{1}{2}h\right) \cdot \left(\vec{\rho}^{(\beta)} - \vec{\rho}^{(\alpha)}\right)\right) + O\left(h^2\right) \qquad (24) \\ &= \sum_{\beta\neq\alpha} \left[\vec{F}_{\beta\alpha}\left(\vec{\rho}^{(\beta)} - \vec{\rho}^{(\alpha)}\right) + \frac{1}{2}\left(h \cdot \nabla\right) \vec{F}_{\beta\alpha}\left(\vec{\rho}^{(\beta)} - \vec{\rho}^{(\alpha)}\right)\right] + O\left(h^2\right) \end{aligned}$$

We will see that this equation for the deviation is identical with that derived for the motion in the TT system. The reason is that the transformation (**??**) basically agrees with the coordinate transformation from the PFR to the TT system. For the distance we have

$$\vec{r}^i \delta_{ij} \vec{r}^j = \vec{\rho}^i \left(\delta_{ik} + \frac{1}{2}h_{ik}\right) \left(\delta_{kj} + \frac{1}{2}h_{kj}\right) \vec{\rho}^j = -\vec{\rho}^i g_{ij}^{TT} \vec{\rho}^j + O\left(h^2\right)$$

and thus all potential forces acting on the deviation of the tidal motion in the PFR system are identical with that in the TT system.

5.3 Hamiltonian analysis in the TT system

We make a potential approximation for the many-particle Hamiltonian, since in a fully relativistic approach we had to include necessarily the dynamics of the electromagnetic field in order to preserve energy-momentum conservation. Thus we write

the total Hamiltonian for many particles with coordinates $\left(x^{(s)\mu}, p_\mu^{(s)}\right)$, $s = 1, 2, ...$ as

$$H = \sum_s H^{(s)} \tag{25}$$

where

$$H^{(s)} = -\frac{1}{2m_1} p_i^{(s)} g^{ij}\left(ct - z^{(s)}\right) p_j^{(s)} + \sum_{s\prime\neq s} \frac{1}{2} V_{ss\prime}(\vec{r}^{(s)} - \vec{r}^{(s\prime)}, ct - z^{(s\prime)}) \tag{26}$$

is the contribution from a single particle. $V_{ss\prime}$ is the total potential generated by the particle s', acting on particle s. As it should be, to each particle only half the potential energy is attributed. In the electromagnetic case, the other half, as well as the infinite self-energy is subtracted with the contribution from the electromagnetic field energy [6]. We assume that the potential is a central potential depending only on the metric distance:

$$V_{ss\prime}(\vec{r}^{(s)} - \vec{r}^{(s\prime)}, ct - z^{(s\prime)}) = V^0_{ss\prime}(\sqrt{-\left(\vec{r}^{(s)} - \vec{r}^{(s\prime)}\right)^i g_{ij}\left(ct - z^{(s\prime)}\right)\left(\vec{r}^{(s)} - \vec{r}^{(s\prime)}\right)^j}),$$

as derived for the Coulomb potential.

The evolution parameter is the time t, common to all particles. The Hamiltonian (25) conserves the difference between the total energy and the center of mass z-momentum:

$$\frac{d}{dt}\left(H + c\sum_s p_3^{(s)}\right) = 0. \tag{27}$$

But since we have less conservation laws than coordinates, energy and momentum transfer from the wave to the particle system has become possible. The change of the total energy is calculated using the equations of motion

$$\begin{aligned} \frac{d}{dt}p_z^{(s)} &= -\frac{\partial H}{\partial z^{(s)}} \\ &= -\frac{1}{2m_s} p_i^{(s)} h^{ij\prime}\left(ct - z^{(s)}\right) p_j^{(s)} \\ &\quad -\frac{1}{2}\frac{\partial}{\partial z^{(s)}}\left(\sum_{s\prime\neq s} V_{s\prime s}(\vec{r}^{(s)} - \vec{r}^{(s\prime)}, ct - z^{(s)}) + \sum_{s'\neq s} V_{ss\prime}(\vec{r}^{(s\prime)} - \vec{r}^{(s)}, ct - z^{(s\prime)})\right) \end{aligned} \tag{28}$$

that lead us to

$$\begin{aligned} \frac{d}{dt}\sum_s p_z^{(s)} &= -\sum_s \frac{1}{2m_s} p_a^{(s)} h^{ab\prime}\left(ct - z^{(s)}\right) p_b^{(s)} \\ &\quad +\frac{1}{2}\sum_{s,s'\neq s} \partial_2 V_{s\prime s}(\vec{r}^{(s)} - \vec{r}^{(s')}, ct - z^{(s)}) \end{aligned} \tag{29}$$

where $h^{ij\prime}$ denotes the derivative, $\partial_2 V_{s\prime s}$ the partial derivative with respect to the second argument only. The derivatives of $V_{s\prime s}$ with respect to the first argument cancel in the sum because of the dependence on $\vec{r}^{(s)} - \vec{r}^{(s')}$ only. To first order in the perturbation we can approximate

$$\partial_2 V_{s\prime s}(\vec{r}^{(s)} - \vec{r}^{(s')}, ct - z^{(s)}) \simeq -V^{0\prime}_{s\prime s}\left(r_{ss\prime}\right)\frac{x^a_{ss\prime}x^b_{ss\prime}}{2r_{ss\prime}} h'_{ab}\left(ct - z^{(s)}\right) \tag{30}$$

where we use $x^a_{ss\prime} = \left(\vec{r}^{(s)} - \vec{r}^{(s\prime)}\right)^a$ for short, and $r_{ss\prime}$ is the unperturbed Euclidean distance. Thus we have (with $h^{ab} = -h_{ab}$ in lowest order)

$$\frac{d}{dt}\sum_s p_z^{(s)} = \sum_s h'_{ab}\left(ct - z^{(s)}\right)\left[\frac{1}{2m_s}p_a^{(s)}p_b^{(s)} - \sum_{s'\neq s} V^{0\prime}_{s\prime s}\left(r_{ss\prime}\right)\frac{x^a_{ss\prime}x^b_{ss\prime}}{2r_{ss\prime}}\right]. \tag{31}$$

For a pulse of duration T that interacts with the particle system the change of the center of mass z-momentum is given by

$$\Delta P_z = \sum_s \int_{t_0}^{t_0+T} h'_{ab}\left(ct - z^{(s)}\right) q^{(s)}_{ab}(t)dt \tag{32}$$

where $q^{(s)}_{ab}$ is the microscopic contribution from a single particle:

$$q^{(s)}_{ab}(t) = \frac{1}{2m_s}p_a^{(s)}p_b^{(s)} - \sum_{s\prime\neq s} V^{0\prime}_{s\prime s}\left(r_{ss\prime}\right)\frac{x^a_{ss\prime}x^b_{ss\prime}}{2r_{ss\prime}}. \tag{33}$$

This contribution is quadrupole-like, but it is weighted with derivative of the potential. Then the sum or the integral over all particles leads in the case of the Coulomb potential to alternating sums where contributions cancel, similar to the summation leading to the Madelung constant. On the other hand, if the potential is purely attractive, as for gravitation, this effect does not occur, giving rise to a completely different response, as well will see.

For a small detector we can assume that the wave field is constant over the particle system represented by the center-of-mass coordinate $z(t)$, so we can change the integration variable to $\tau = t - z(t)/c$ and obtain

$$\Delta P_z = \int_{-\infty}^{+\infty} d\tau h'_{ab}\left(c\tau\right)\sum_s \frac{q^{(s)}_{ab}(t(\tau))}{1-\dot{z}(t(\tau))/c} = \int_{-\infty}^{+\infty} d\tau h'_{ab}\left(c\tau\right) Q_{ab}(\tau) \tag{34}$$

with

$$Q_{ab}(\tau) = \sum_s \frac{q^{(s)}_{ab}(t(\tau))}{1-\dot{z}(t(\tau))/c}. \tag{35}$$

For a wave pulse or wave train with duration T, such that $h_{ab}\left(0\right) = h_{ab}\left(cT\right) = 0$ we use partial integration to write (34) as

$$\Delta E = c\Delta P_z = -\int_{-\infty}^{+\infty} d\tau h_{ab}\left(c\tau\right) Q'_{ab}(\tau) \tag{36}$$

Thus the response of the detector to a gravitational wave pulse travelling in z-direction is described by the time-dependence of the microscopic function Q_{ab}. Alternatively, we may use Fourier transform to arrive at

$$\Delta E = \int_{-\infty}^{+\infty} d\nu \hat{h}^*_{ab}(\nu) i\nu \hat{Q}_{ab}(\nu) \tag{37}$$

with the spectral decompositions of the wave:

$$\hat{h}_{ab}(\nu) = \int h_{ab}(c\tau)e^{-2\pi i\nu\tau}d\tau, \tag{38}$$

and the particle system:

$$\hat{Q}_{ab}(\nu) = \sum_s \int \frac{q^{(s)}_{ab}(t(\tau))}{1-\dot{z}(t(\tau))/c}e^{-2\pi i\nu\tau}d\tau. \tag{39}$$

Thus $\hat{Q}_{ab}$ represents the effective cross section of the particle system on the microscopic level.

6 Force distribution in a massive body

We first consider electromagnetically coupled bodies, as ion crystals and metals, where the gravitational forces between the constituents play no role. Ion crystals have the advantage that all charges can be considered to be point-like and located on lattice points. Thus only discrete sums have to be evaluated. As we have seen, the forces driving the deviations from the tidal motion in the PFR system are identical to the forces driving the whole motion in the TT system as long as we consider only motions of order $O(h)$.

6.1 Ion crystal

We first consider a gravitational wave propagating in z-direction incident on a lattice of ions. In order for the system to possess a stable ground state, we have to include not only the Coulomb potential, but also some (short-ranged) repulsive potential. The Coulomb force exerted by particle $s\prime$ on particle s is given, to first order in h, by

$$F^a_{C,\, ss\prime} = \frac{q_s q_{s\prime}}{4\pi\ \varepsilon_0}\left(\frac{x^a_{ss\prime}}{r^3_{ss\prime}} - h_{ab}\frac{x^b_{ss\prime}}{r^3_{ss\prime}} + \frac{3}{2}h_{bc}\frac{x^b_{ss\prime}x^c_{ss\prime}x^a_{ss\prime}}{r^5_{ss\prime}}\right), \tag{40}$$

for the x- and y- directions, the force in z-direction additionally involves the derivative of h, which is not of interest here. In the case of the Born-Meyer potential the repulsive potential is of exponential form

$$V_{BM}(r_{ss\prime}) = A_{ss\prime}e^{-\beta r_{ss\prime}} \tag{41}$$

with couplings $A_{ss\prime}$ that depend on the charges. Though it is necessary to include this potential, its precise form is not crucial for our further considerations. We obtain an additional force from this potential, up to first order in h given by

$$\begin{aligned} F^a_{BM,ss\prime} &= -\frac{\partial V_{BM,}}{\partial x^{a\ (s)}}, \\ &= A_{ss\prime}\beta\frac{e^{-\beta r_{ss\prime}}}{r_{ss\prime}}\,x^a_{ss\prime} + \frac{1}{2}\,A_{ss\prime}\beta\,\frac{e^{-\beta r_{ss\prime}}}{r_{ss\prime}}\,\left(\frac{1}{r^2_{ss\prime}} + \beta\frac{1}{r_{ss\prime}}\right)h_{bc}\,x^b_{ss\prime}\,x^c_{ss\prime}\,x^a_{ss\prime} \\ &\quad -A_{ss\prime}\beta\,\frac{e^{-\beta r_{ss\prime}}}{r_{ss\prime}}\,h_{ab}\,x^b_{ss\prime}. \end{aligned} \tag{42}$$

The total force on ion s is given by

$$\begin{aligned} F^a_s &= \sum_{s\prime\neq s}\left(\frac{q_s q_{s\prime}}{4\pi\varepsilon_0}\,\frac{1}{r^3_{ss\prime}} + A_{ss'}\beta\frac{e^{-\beta r_{ss\prime}}}{r_{ss\prime}}\right)\,x^a_{ss\prime} \\ &\quad -h_{ab}\,\sum_{s\prime\neq s}\left(\frac{q_s q_{s\prime}}{4\pi\varepsilon_0}\,\frac{1}{r^3_{ss\prime}} + A_{ss\prime}\beta\frac{e^{-\beta r_{ss\prime}}}{r_{ss\prime}}\right)\,x^b_{ss\prime} \\ &\quad +h_{bc}\,\sum_{s\prime\neq s}\left(\frac{3q_s q_{s\prime}}{8\pi\varepsilon_0}\,\frac{1}{r^5_{ss\prime}} + \frac{1}{2}A_{ss\prime}\beta\frac{e^{-\beta r_{ss\prime}}}{r_{ss\prime}}\left(\frac{1}{r^2_{ss\prime}} + \frac{\beta}{r_{ss\prime}}\right)\right)\,x^b_{ss\prime}\,x^c_{ss\prime}\,x^a_{ss\prime} \end{aligned} \tag{43}$$

Assuming that the ion chain is in its unperturbed equilibrium state, both the first and second terms vanish because the vanishing of the first term defines the equilibrium in absence of a gravitational wave, and the second term is just the first

multiplied by the matrix h_{ab}. Thus the gravitational wave gives rise to a perturbation of the equilibrium state induce by the force

$$\Delta F_s^a = \left[h_{bc} \sum_{s\prime \neq s} \left(\frac{3 q_s q_{s\prime}}{8\pi\varepsilon_0} \frac{1}{r_{ss\prime}^5} + \frac{1}{2} A_{ss\prime} \beta \frac{e^{-\beta r_{ss\prime}}}{r_{ss\prime}} \left(\frac{1}{r_{ss\prime}^2} + \frac{\beta}{r_{ss\prime}} \right) \right) x_{ss\prime}^b \, x_{ss\prime}^c \right] x_{ss\prime}^a \tag{44}$$

Once the system is driven out of the equilibrium state, we still can ignore the second term in (43), because for deviations $\Delta x^a_{ss\prime} \sim h$ from equilibrium this term is only of order h^2. The first term then describes phonon-graviton interaction.

The sum over all other ion in (44) now depends on the dimension of the lattice. The sum over the short-ranged part converges even for an infinite lattice, so that its contribution is always limited. The sum over the Coulomb part is an alternating sum. In one dimension, this sum is always bounded by the first term that is not canceled by a contribution from a symmetric neighbor, thus the force has a maximum on the endpoints and decreases proportional to R^{-2} with the distance R from the endpoints. In two dimensions we find that the force distribution exhibits the correct quadrupole structure [8]. Therefore acoustic modes are excited. This result is obtained only when the short-range potential is included. The Coulomb forces alone gives rise to forces that additionally alternate in direction from ion to ion, so that we would have arrived at the wrong conclusion that optical modes are excited.

The forces decay rapidly away from the boundary and do not follow the linear law of the tidal accelerations. Thus, the deviations from the tidal motion excite a broad spectrum of modes, of which few are resonantly driven. We now present a general argument that this pertains to the relevant case of three dimensions.

6.2 General case

Since the temperature of the body must be low in order to achieve the desired sensitivity of a detector, we assume that the mass is a perfect crystal, either an ion crystal with discrete charges locate on some lattice, or a metal with ions on some lattice and the electron gas in between. The body is decomposed into a finite number of elementary cells V_i with their charge centers at $\vec{r}_i$, that are (i) electrically neutral,

$$\int_{V_i} d^D r \rho(\vec{r}) = 0 \tag{45}$$

and (ii) do not possess an electric dipole moment,

$$\int_{V_i} d^D r \, (\vec{r} - \vec{r}_i) \, \rho(\vec{r}) = 0. \tag{46}$$

D is the spatial dimension of the lattice. We now consider a charge element $q' = \rho(\vec{r}\prime)dV$ located at $\vec{r}\prime$ and the perturbational force exerted on it by the elementary cell V located at $\vec{r}_i = 0$, according to (44). For $|\vec{r}\prime| \gg 1/\beta$ we can ignore the short-range potential and have

$$F_s^a \left(\vec{r}\prime\right) = q' \int_V d^D r \; \frac{3\rho(\vec{r})}{8\pi\varepsilon_0} \, \frac{\left(\vec{r}\prime - \vec{r}\right)^a}{\left|\vec{r}\prime - \vec{r}\right|^5} h_{bc} \, \left(\vec{r}\prime - \vec{r}\right)^b \left(\vec{r}\prime - \vec{r}\right)^c. \tag{47}$$

We expand the integrand into powers of $\vec{r} = (\ x, y, z)$ up to second order. Then the integrals of the first and second order vanish due to conditions (45) and (46),

respectively. As an example, if h is of +-polarization, the forces in two dimensions are given by

$$F^{x}_{q',V}(\vec{r}') = q'h_{+}\left[\frac{x'\left(6x'^2-9y'^2\right)}{2r'^7}I_1+\frac{x'\left(-3x'^2+12y'^2\right)}{2r'^7}I_2+\frac{y'\left(12x'^2-3y'^2\right)}{r'^7}I_3\right] \tag{48}$$

$$F^{y}_{q',V}(\vec{r}') = q'h_{+}\left[\frac{y'\left(12x'^2-3y'^2\right)}{2r'^7}I_1+\frac{y'\left(-9x'^2+6y'^2\right)}{2r'^7}I_2+\frac{x'\left(-3x'^2+12y'^2\right)}{r'^7}I_3\right] \tag{49}$$

where the integrals

$$I_1=\int_V d^D r\, x^2\, \frac{3\rho(\vec{r})}{8\pi\varepsilon_0},\quad I_2=\int_V d^D r\, y^2\, \frac{3\rho(\vec{r})}{8\pi\varepsilon_0},\quad I_3=\int_V d^D r\, xy\, \frac{3\rho(\vec{r})}{8\pi\varepsilon_0} \tag{50}$$

describe the quadrupole moments of the electric charge distribution in the elementary cell. The generalization to three dimensions is straightforward, with the same qualitative properties: The force exerted by some elementary cell on a charge element q' always decreases at least proportional to r'^{-4} with the distance between the charge and the cell. If the quadrupole moments vanish, as for cubic lattices, the decrease is even faster by two powers of r'.This implies that for a given charge element q' the sum over all elementary cells always converges in $D = 1, 2$, or 3 dimensions. Therefore the force on any ion in the body is bounded independently from the size of the body,

$$\left|\vec{F}_{q'}\right|_{\max} \le \sum_i \left|F^{y}_{q',V_i}(\vec{r}'-\vec{r}_i)\right| \le const \sum_i \frac{1}{\left|\vec{r}'-\vec{r}_i\right|^4}. \tag{51}$$

Further, the reflection symmetry of the lattice implies that all forces from cells in a volume that possesses reflection symmetry around the charge element add up to zero. Hence also in the general case the charge element feels a force that depends on its distance R to the surface of the body. A crude estimate gives

$$\left|\vec{F}_{q'}\right|_{R} \le \sum_{i,\left|\vec{r}'-\vec{r}_i\right|>R} \left|F^{y}_{q',V_i}(\vec{r}'-\vec{r}_i)\right| \le const \sum_{\left|\vec{r}'-\vec{r}_i\right|>R} \frac{1}{\left|\vec{r}'-\vec{r}_i\right|^4} \sim R^{D-4}. \tag{52}$$

Depending on the dimension, we observe that the force decreases with the distance from the surface, at least with $1/R$ in three dimensions, if the quadrupole moments I_1, I_2, I_3 all vanish the decrease is even stronger. Scaling with the lattice constant a leads us to

$$\left|\vec{F}_{q'}\right|_{R} \le \left|\vec{F}_{q'}\right|_{\max}\left(\frac{a}{R}\right)^{4-D}. \tag{53}$$

Because the sum over the forces from the short range potential decreases even faster, the total perturbational forces, which are maximal on the surface, decrease to about 10^{-3} of the surface value within 1000 atomic layer, which is about 1micrometer. When we integrate the forces (48,49) over an elementary cell in order to obtain the mean force on the cell, we again loose two powers due to (45) and (46), resulting in mean forces between two cells V_i and V_j that decrease at best proportional to $|\vec{r}_j-\vec{r}_i|^{-6}$. Thus we conclude that the bulk of the material remains, apart from tidal motion, unaffected by the forces induced by the gravitational wave. This result has its origin in the nature of the electromagnetic coupling with its charges of different signs, and the structure of solid bodies with reflection symmetry of the elementary cells.

6.3 Gravitationally coupled matter

Clearly, the result of the preceding section was due to the conditions (45,46), but (45) does not hold for the attractive gravitational forces. The acceleration of a test mass is given by

$$\vec{a}_G\left(\vec{r}'\right) = \int_V d^D r \; \frac{\rho_m(\vec{r})}{G} \; \frac{\left(\vec{r}' - \vec{r}\right)}{\left|\vec{r}' - \vec{r}\right|^5} h_{bc} \; \left(\vec{r}' - \vec{r}\right)^b \left(\vec{r}' - \vec{r}\right)^c , \tag{54}$$

where ρ_m is the mass density of the body. Assuming that the wavelength of the gravitational wave is large compared to the dimensions of a homogeneous massive body, we can take ρ_m and h to be constant and are able to evaluate the integral for simple geometries. For a $+$-polarized wave propagating in z-direction the maximal acceleration on the surface is calculated to

$$\left|\vec{a}_G\right|_{\max} = \begin{cases} \rho_m \pi r \left(1 - \frac{1}{\sqrt{1+\frac{l^2}{r^2}}}\right) & \text{for a cylinder of radius } r \text{ and lenght } l \\ \rho_m \pi \frac{\tan^2\phi}{(1+\tan^2\phi)^{3/2}} l & \text{for the tip of a cone of opening angle } \phi \text{ and height } l \\ \rho_m \pi \frac{8R}{15} & \text{at the surface a sphere of radius } R. \end{cases} \tag{55}$$

Naturally, there exists an angular dependence of the forces of quadrupole characteristic. Thus the forces grow linearly with the linear dimensions of the massive body. The maximal acceleration on the equator of a sphere is given by

$$\left|\vec{a}_G\right|_{\max}^{TT} = \frac{2}{5}\left|h\right| g \tag{56}$$

where g is the surface acceleration of the mass. This force will truly excite the quadrupole modes in the bulk of a body and is able to do work against the gravitational and electromagnetic forces that keep the body together. Comparing this with the tidal acceleration in the PFR system,

$$\left|\vec{a}_G\right|_{\max}^{PFR} = \frac{1}{2}\omega^2 R \left|h\right| , \tag{57}$$

we see that (56) is several orders of magnitude smaller than (57). In the PFR system the forces drive primarily the tidal motion, only accelerations of order (56) drive the deviations.

7 Detector Types

7.1 Rotational Detectors

From the structure of the microscopic quantity $Q_{ab}(\tau)$ we can immediately identify the different types of detectors. If the body is rotating relative to the TT frame with some frequency ω, then the momentum contribution

$$\sum_s \frac{1}{2m_s} p_a^{(s)} p_b^{(s)} \sim \omega^2 \cos 2\omega t \tag{58}$$

dominates and gives rise to a response of order h. This type of detector was first suggested by Braginsky [9, 2]. We estimate the response of a rotating mass to the gravitational wave using our model. From (34) we obtain a momentum input to the center of mass of

$$dP_z/dt = h_0 \cos\left(\omega_0 t + \varphi\right) \cos 2\omega t \frac{\omega^2 \omega_0 l^2 M}{24c} \tag{59}$$

where l is the length of the bar, M its mass, and h_0, ω_0, and φ are the amplitude, frequency and phase difference of the gravitational wave, respectively. This is for the case where the wave vector is perpendicular to the plane of rotation. In the ideal case $\omega \approx \omega_0/2, \varphi = 0$ or π, the mean energy input is given by

$$\dot{E} = \pm h_0 \frac{\omega_0^3 l^2 M}{192}. \tag{60}$$

Note that the change can be of either sign, thus the gravitational wave cannot only be absorbed, but can also stimulate the emission of gravitational waves from the system. For reasonable values (bar of $1m$, $1-100kg$ mass, $\omega/2\pi \sim 10-1000$ Hertz, $h_0 \sim 10^{-20}$) the attainable energy input ranges in about

$$\dot{E} \sim 10^{-13}...10^{-9}W. \tag{61}$$

Though this is quite large compared to the resonant detector, it seems questionable whether this change in the rotational energy of the bar can be measured. Certainly the acceleration of the rotating bar in the direction of the wave, given by (59), is too small to be detectable. This gives additional justification to the negligence of the acceleration of the center of mass in the transformation to the PFR system [2].

7.2 Resonant Detectors

For a non-rotating mass, the lowest order contribution stems from the time derivatives of the positions and momenta induced by the wave and thus leads to a response of order h^2. This is the Weber detector. Our considerations have shown that the energy input into such a detector occurs primarily at its surface. This implies that when resonance is discussed, we have to take the time into account that is needed to transport form the surface to the interior of the body. For a material with high Q-factor the velocity of transport is given by the velocity of sound. Thus it takes a time

$$T_v = \frac{L}{2v_s} \tag{62}$$

where L is the diameter of the body and v_s the velocity of sound before energy reaches the center of the body. This time limits the onset of resonance. For example, for GRAIL [?] with diameter $L = 3m$, $v_s \approx 4000m/s$ we have $T_v \approx 3/8$ milliseconds. Thus this time scale will play a role for the detection of millisecond pulses. In order to arrive at the resonant amplitude, pulses must be considerably longer than T_v. How many oscillations are needed to arrive at the maximal resonant amplitude, can also be estimated in the normal-mode model.

In general, if a weakly damped oscillator characterized by a frequency ω_0 is excited by a force $F = \varepsilon\omega^2 sin\,(\omega t)$ near resonance, $\omega \approx \omega_0$, then if the oscillator is initially at rest, the subsequent maxima of the amplitude of the oscillator will at the beginning follow a linear law

$$|A_{max}| \sim \frac{\varepsilon\omega^2}{2\omega_0} t \tag{63}$$

until the resonant amplitude of order $C = \varepsilon C_0$ is reached after a transient time

$$T_{trans} \approx \frac{2C_0}{\omega} \approx \frac{2C_0}{\omega_0}. \tag{64}$$

Thus e.g. it takes for a signal of frequency $\omega = 1000Hz$ about $2s$ until it is enhanced by a factor $C_0 = 1000$. Thus 2000 oscillations are needed before the maximal

resonant level is reached. In general the effective cross section of a resonant mass detector is proportional to $(C_0)^2$, but for pulses shorter than T_{trans} an additional factor $(T/T_{trans})^2$ must be taken into account. This is relevant to the detection of millisecond pulses with a detector like GRAIL that operates at a fundamental frequency of $650Hz$.

A further point to consider certainly is the impurity of the material. When grains of material stick together, we also have to consider internal boundary that behave like surfaces. But because this results in random effects, the impurity will hardly improve the behavior of the detector.

7.3 Microscopic Detector

We have shown that in the TT system the metric effects on the electromagnetic coupling cancel in the bulk of a massive body, in the PFR system the deviations from the tidal motion are driven only at the surface. This raises the question whether this type of surface effect could be used in some other way for detection of gravitational waves. Similar to expression (56) for the gravitationally coupled matter, we can estimate the acceleration induce by the gravitational wave on the surface of an ion lattice by

$$|\vec{a}_{EM}|^{TT}_{\max} \sim \left| \frac{Z_+ Z_-}{m_\pm d^2 \pi \varepsilon_0} h \right| \tag{65}$$

where Z_+, Z_- and m_+, m_- are the charges and the masses of the ions, respectively, and d the lattice constant. A geometric factor of order 1 has to be included in addition. This factor will depend on the structure of the lattice and the precise behavior of the repulsive potential. The numerical value of (65) is of the order of $h \cdot 10^{16} ms^{-2}$ (for KCl) which is several orders of magnitude larger than (57). This suggests that the piezoelectric effect might be used for experimental verification, in a similar way like a pressure sensor works. For h in the order of 10^{-22} accelerations are of order $10^{-6} ms^{-2}$ and possibly are within the reach of sensitive detectors. If we consider a piezo crystal under strain our analysis has to be revised. Because we then have a dipole moment in each elementary cell, the leading order of the forces will be proportional to r'^{-3} as compared to r'^{-4} in (48) and (49). Due to the uniform direction of the dipoles and the broken refection symmetry the contributions in the integration will no longer cancel, so we expect that the forces grow logarithmically with the dimension of the crystal and the excitation occurs truly throughout the bulk. We hope to be able present a detailed analysis of this case, together with possible detector design, soon elsewhere.

8 Conclusions

We analyzed the resonant mass gravitational wave detector form a microscopic point of view, using the wave guide (TT) frame of reference along with the conventional PFR system. In the TT system the variation of the Coulomb field of the constituent charges of the body gives the dominant contribution, the resulting forces agrees with those driving the deviation from the tidal motion in the PFR system. For an electromagnetically coupled body with reflection symmetry, this force distribution is such that the forces are restricted to a small surface layer, with no bulk force. Hence the relevant energy input occurs at the surface only. This effect has its origin in the fact that the pattern of the tidal forces comes close to a null mode of the system, not doing work against the coupling forces, as we saw from a linear chain model. This result is not in contradiction to the standard normal mode analysis, rather it reflects a local property of the response compared to global nature of normal

modes. For the gravitationally coupled body, we observe a linear force law and bulk excitation, but the part that causes resonant response is orders of magnitude smaller than expected from the tidal forces in the PFR system.

Regarding resonant mass detectors, the local nature of the driving force makes it necessary to consider the time scale on which the energy is transported into the interior. This time scale is set by the velocity of sound. As a result, it might take too long a time for a resonant detector to be excited measurably by short millisecond pulses as are emitted by collapsing stellar objects.

Finally, we presented ideas for a new type of microscopic detector that employs the piezo effect. The force pattern on a crystal is basically that of a quadrupole-like distributed pressure change, the surface force is independent of the size of the crystal. For a crystal under strain the force even acts throughout the bulk. An analysis how the desired sensitivity can be achieved was outside the scope of the work presented here.

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